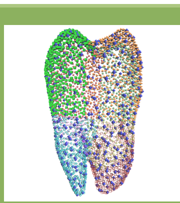

Hierarchically Partitioned Implicit Surfaces For Interpolating Large Point Set Models

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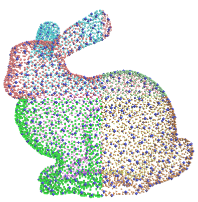
We present a hierarchical spatial partitioning method for creating interpolating implicit surfaces using **compactly supported radial basis functions** (RBFs) from scattered surface data. From this hierarchy of functions we create a range of models from coarse to fine, where a coarse model approximates and a fine model interpolates all the surface points.

As with other hierarchical partitioning approaches, our method builds a large-scale approximating embedding function then successively refines it with smaller-scale incremental functions. The two main components of our method are selecting the points for each node in the hierarchy and creating phantom constraints to handle overlapping function domains. Using phantom constraints to clamp each embedding function allows us to combine them simply by addition rather than requiring a blending function.

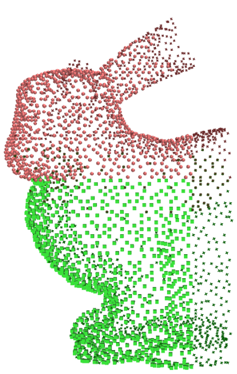
Hierarchy

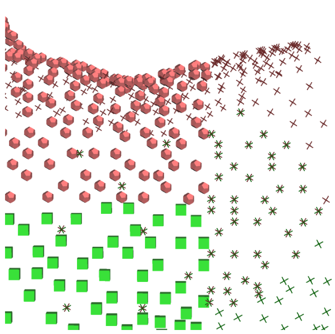


To build a hierarchy we use an octree to span the input points, which is traversed from the top down. Points are first selected for the root, producing an embedding function for a base model. Then points for the each of the children of the root are selected, adding detail at a finer resolution. After solving the refining embedding functions for the eight children, we proceed to the grandchildren, and so on, stopping when all points have been included in the hierarchy.

Points are selected in a node's octant based on a random Poisson-disk distribution. We use a **modified distance function** to allow denser sampling in regions of high curvature. Our distance function is the Euclidean distance scaled by a polynomial based on the disparity between the points' normal orientations.

For the root node of the hierarchy we only select points that are "representative" of a local region. The goal is to pick points that capture the larger-scale shape of a region, pushing smaller scale detail or noise to nodes lower in the hierarchy. For the root node, candidate points are screened by comparing the normal direction for each point with the normals of points around it and rejecting those that are too disparate.





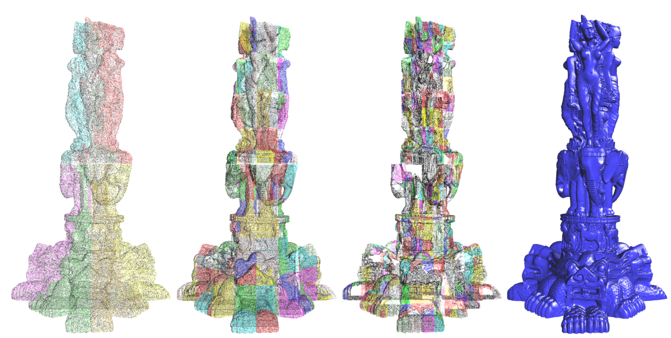
Phantom Constraints

Managing embedding functions with overlapping extent is a common problem that occurs when attempting to partition a point set. To interpolate all the points in a data set, we must guarantee that the combination of all embedding functions that impinge on a point produces the exact value we require. Our task is simplified by the compact RBF's limited extent. Therefore at any given point only relatively few embedding functions need to be combined and evaluated.

Our approach is to place **phantom constraints** in a given node to **clamp its embedding function**. Phantom constraints are placed in regions where the extent of the embedding functions for different nodes overlap, requiring us to suppress the influence of the node's embedding function. In this way, phantom constraints serve much the same purpose as the blending function in partition-of-unity approaches but without explicit blending during evaluation of the implicit surface's embedding function. The locations for phantom constraints fall into two categories: locations that have been inherited from regular constraints in ancestral nodes, and locations from regular constraints in adjacent sibling nodes. Using a top-down approach means constraint locations from descendant nodes can be ignored.

Results

We applied our method to the Thai statue from the Stanford 3D Scanning Repository. The image below shows the 1st, 3rd, and 4th hierarchy levels and an extracted iso-surface. The table below shows the statistics for the model, for each implicit evaluation and the error for all the surface points.



		Thai statue
Hierarchy statistics	*vertices*	4,999,996
	regular constraints	10,852,316
	phantom constraints	6,632,801
	total constraints	17,485,117
	tree nodes	799
	tree height	5
	build time (minutes)	127.35
Averages per evaluation	*regular constraints*	961.77
	phantom constraints	202.00
	number of nodes	6.59
Error	*avg. implicit error*	1.4980E-06
	max. implicit error	1.4950E-04
	avg. distance error	2.1230E-10
	max. distance error	2.2700E-08

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